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TECHNICAL NOTE

REFERENCE FRAMES, MODELS AND CONVENTIONS FOR ROSETTA LANDER GROUND SEGMENT

Written by : SONC Flight Dynamics Team	Date :	
Checked by : BIELE Jens DLR/MUSC MAIBAUM Michael DLR/MUSC	Date :	
Approved by : GAUDON Philippe DCT/ME /EU	Date :	

STFC

INDEX SHEET

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TITLE :

TECHNICAL NOTE

REFERENCE FRAMES, MODELS AND CONVENTIONS FOR ROSETTA LANDER GROUND SEGMENT

AUTHOR(S) :

SONC Flight Dynamics Team

SUMMARY : This technical note defines the reference frames and conventions needed for the philae / Rosetta mission.

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ACCOMAZZO Andréa	ESA/ESOC (andrea.accomazzo@esa.int)	
HERFORT Ulrich	ESA/ESOC (ulrich.herfort@esa.int)	
Philae Lander Control Centre	lcc@dlr.de	

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01	07	14/10/2013	intentionally empty SONC Flight Dynamics Team Corrections in 7.2.6 Transformations from LSF to CFF Chapter 7.2 Local Orbital Comet frame move to annexe A.4 Local Orbital frames Correction in 6.3.2 Transformation from LGF to LDR typo BDT was used instead of TDB Update of RD2, RD4 and RD6
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01	05	23/04/2012	intentionally empty SONC Flight Dynamics Team Change in explanatory text of figure 6. Figure 9 replaced.
01	04	05/04/2012	intentionally empty SONC Flight Dynamics Team After check by J. Biele, M. Maibaum, K. Geurts. Main changes: Addition of the Solar Aspect Angles to irradiation angles transformation. Explicit transformation from QSW to EME2000. Modification of the summary table (Introduction).
01	03	27/03/2012	intentionally empty SONC Flight Dynamics Team After check by J. Biele, M. Maibaum, K. Geurts. Main changes: Removal of unnecessary sections (Brillouin sphere, geodetic coordinates...) and obsolete reference documents. Addition of summary table. Addition of the LSF to CFF transformation.
01	02	09/03/2012	intentionally empty SONC Flight Dynamics Team Several coordinate changes added: from Rosetta S/C frame to Philae Lander frame, from Philae Lander Frame to Landing Gear frame and from Local site reference frame to Landing Gear frame.
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01	00	22/10/2009	intentionally empty

			MARTIN Thierry BLAZQUEZ Alejandro Creation of the document	DCT/SB/MS DCT/SB/MO

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GLOSSARY AND LIST OF TBC AND TBD ITEMS

CFF	Comet Fixed Frame
CNES	French National Space Agency
EME2000	Earth Mean Equator and Equinox of epoch J2000
ESA	European Space Agency
ESOC	European Space Operation Center
ET	Ephemeris Time
FSS	First Science Sequence
LDR	Lander Frame
LG	Landing Gear
LGF	Landing Gear Frame
LSF	Landing Site Local Frame
LTS	Long Term Science
QSW	Local Orbit Frame
RGS	Rosetta Ground Segment
RLGS	Rosetta Lander Ground Segment
SAS	Solar Array Simulator
SDL	Separation, Descent and Landing
TAI	Temps Atomique International
TDB	Barycentric Dynamical Time
URF	Unit Reference Frame
UTC	Universal Time Coordinated
w.r.t.	with respect to
WGS	World Geodetic System

List of TBC items :

List of TBD items :

1. OVERVIEW

1.1. REFERENCE DOCUMENTS

RD1 MSLIB Volume R Les repères fondamentaux
CNES MSLIB, 03/03/2009, Issue 6, Rev. 3
BIBMS-mu-292-2012-ATOS

RD2 Lander User Manual
MAIBAUM Michael, 07/09/2012, Issue 5, Rev. 1
RO-DLR-UM-3100

RD3 *Lander User Manual 2.2-9b Solar Arrays*
Michael Maibaum, Lander Control Centre, 16/04/2008, Issue 1
RO-LAN-UM-3100-SA (In Preparation)

RD4 Lander User Manual 2.2-6 Landing Gear
Michael Maibaum, 23/04/2012, Issue 1, Rev. 1
RO-LAN-UM-3100-LG

RD5 Report of the IAU/IAG working group on cartographic coordinates and rotational elements 2009
P.K. Seidelmann and al, 04/12/2010, Issue 1, Rev. 0
IAU-2009

RD6 Data Delivery Interface Document Appendix H FD Products
Rosetta/MEX/VEX Ground Segment Teams, 20/10/2012, Issue 4, Rev. 2
RO-ESC-IF-5003 Appx H

RD7 Spacecraft attitude determination and control, Kluwer Academic Publishers, 1978
James R. Wertz
ISBN-978-9027712042

1.2. APPLICABLE DOCUMENTS

2. INTRODUCTION

This technical note defines the time scales, reference frames and conventions required by the Philae mission. Rotation matrix for transformations between reference frames are provided, whenever necessary and possible with the information which is available at the moment of writing the present document. The goal is that a chain of transformation allows, for instance, the transformation of the coordinates of a vector from camera axis (during descent / cruising phase or landed) into EME2000 (and vice versa). See Table 1 Chains of transformations between different reference frames, at which moment they are needed and in which section of this technical note we can find them

for a summary of which transformations are given explicitly, for which objective they are used as well as where to find them in this technical note.

	CFF	LSF	LGF	LDR	Rosetta
EME2000	ESA CKIN file, 7.1.3	LSF ↔CFF ↔ EME2000	LGF↔LDR↔ LSF ↔ CFF↔ EME2000	LDR ↔ LSF ↔ CFF↔ EME2000	ESA Rosetta attitude file
CFF		7.2.6 Computation of events, masking…	LGF↔LDR ↔LSF↔CFF Computation of optimal attitude for illumination during FSS and LTS	LDR ↔ LSF↔ CFF Attitude determination after landing	
LSF			LSF↔ LDR ↔ LGF Attitude determination and computation of optimal attitude for illumination during FSS and LTS	7.2.4 Attitude determination after landing	
LGF				6.3.4 Computation of optimal attitude after landing	
SAA				6.2.2 Computation of Solar Power before SDL	6.2.2.1 Computation of Solar Power before SDL
LDR					6.1.3 Computation of the separation conditions

Table 1 Chains of transformations between different reference frames, at which moment they are needed and in which section of this technical note we can find them

When not specified otherwise, the units of the position coordinates are millimetres (mm).

3. TIME DEFINITIONS

The time scale used by the Rosetta project is the **Universal Time Coordinated**, UTC.

The UTC is deduced from "**Temps Atomic International**", TAI, so as not to entirely lose the relation between time and Earth's orientation (UT1).

The TAI is the continuous time scale produced by the Bureau International de l'Heure in Paris. It is a time scale based on the readings of approximately 150 atomic clocks

Since January 1 1972, the UTC and TAI differ by an integer number of seconds, and adjustments are made whenever UTC deviates by more than ± 0.9 second from UT1.

- TAI-UTC=n seconds (n integer ≠0)
- $|$UT1-UTC$|$ < 0.9 seconds

Thus the difference between UTC and TAI is constant over a given period.

The **Ephemeris Time** scale (ET) is a time scale based on the Earth revolution around the Sun. Defined with:

- ET=TAI+32.184 s

The scale **Barycentric Dynamic Time** (BDT) is very close to the Ephemeris time scale. It is assumed for mission analysis purposes that the BDT scale is the same as the ET.

4. CELESTIAL REFERENCE FRAMES

4.1. EARTH MEAN EQUATOR AND EQUINOX OF J2000 (EME2000)

4.1.1. Purpose

The Earth Mean Equator and Equinox of epoch J2000 reference system (also known as EME2000) is the prime coordinate system used to exchange orbital data (state vectors) and attitude data between RLGS and RGS. The comet position data and orientation data will be exchanged using the EME2000 reference frame. The Rosetta spacecraft position and orientation will be exchanged using the EME2000 reference frame.

4.1.2. Definition

This Reference frame is also known as Mean Equator and Equinox. It is a standard inertial Earth centred, equatorial, mean of epoch reference system. It is an inertial Reference frame defined as follows (see RD1 for software implementation):

- The Reference epoch is January 1st, 2000 at 12h00 (TDB time scale, Julian date 2451545.0)
- The origin 0 is the Earth centre of mass of WGS 84 ellipsoid
- $\mathbf{Z}_{2000}$ points along the mean Earth rotational axis at the reference epoch oriented towards the North pole
- $\mathbf{X}_{2000}$ axis points toward the mean vernal equinox of the reference epoch.
- $\mathbf{Y}_{2000}$ axis completes the right handed coordinate system.
- ($\mathbf{X}_{2000}$, $\mathbf{Y}_{2000}$) lies in the mean equatorial plane of the reference epoch

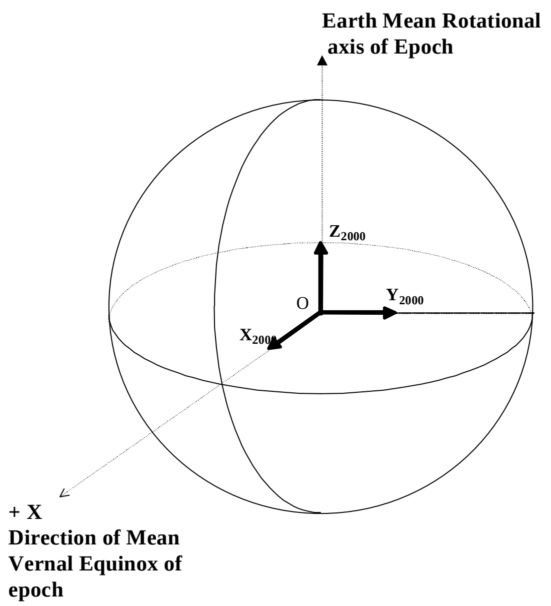


Figure 1 EME2000 axis in inertial space.

5. ROSETTA UNIT REFERENCE FRAME, R_{ORB}

Transformations between Rosetta S/C ORB frame and Philae Lander reference frame are necessary before SDL, at the end of the attached phase. The following figures, describing this transformation, have been taken from *RD2*.

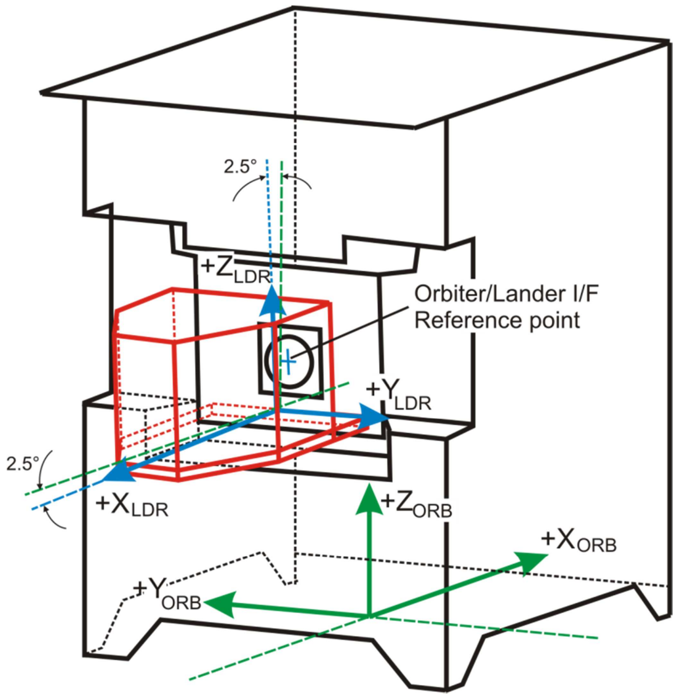


Figure 2 Schema of the Lander and Orbiter Frames in the attached or cruise configuration.

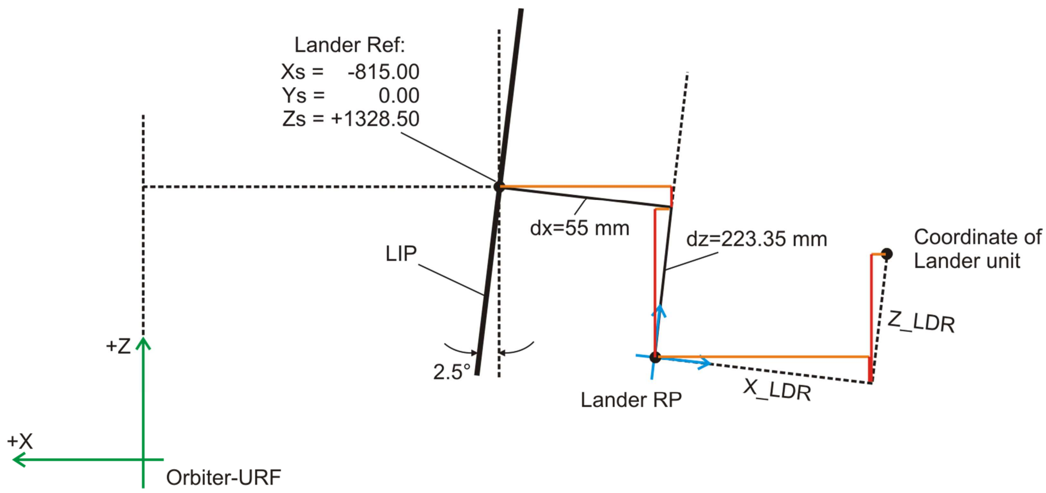


Figure 3 XZ view of the alignment between LDR frame and Rosetta S/C Frame (w.r.t. orbiter URF). See §6.1.3 for the transformation between orbiter and Lander URF)

6. REFERENCE FRAMES RELATED TO THE LANDER

6.1. PHILAE LANDER REFERENCE FRAME - R_{LDR}

6.1.1. Purpose

This is the main frame of the Philae Lander. Lander equipment's and instruments (cameras) positions are defined in this frame.

Thanks to the transformations described in this technical note, it will be possible to give the direction of a camera axis or a solar panel, for example, and to transform them to other frames.

6.1.2. Definition

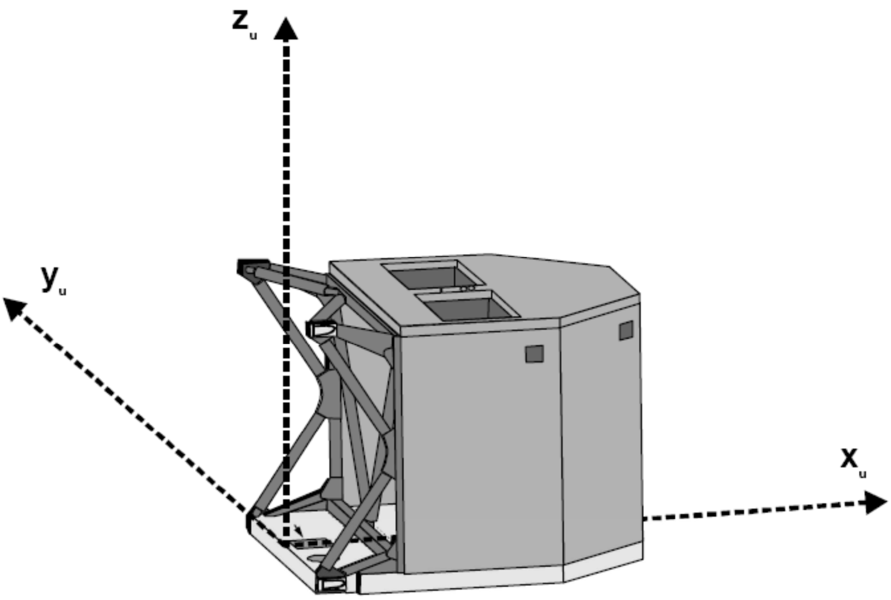


Figure 4 Philae Lander Reference frame (URF).

The Philae Lander Reference Frame is defined as follows (see Figure 4):

- Origin: On the upper surface of the balcony plate, in the middle of the free edge.
- X_{LDR} is pointing at the middle of the wall opposite to the balcony (direction of the Lander separation).
- Z_{LDR} is pointing from the baseplate to the solar hood lid.
- Y_{LDR} axis completes the right handed coordinate system

6.1.3. Transformation from Rosetta S/C to Philae LDR frame

If X_{ORB} are coordinates of a point expressed in Rosetta S/C frame and X_{LDR} the coordinates expressed in Lander Frame, the following formulae apply for the transformations (see Figure 2 and Figure 3).

$$X_{ORB} = \begin{pmatrix} -860.7063 \\ -0.5490 \\ 1103.0652 \end{pmatrix} + \begin{bmatrix} -0.999132123 & 0.004472887 & -0.04141249 \\ -0.004472887 & -0.999983661 & 0.003371339 \\ -0.041396734 & 0.003559591 & 0.999136447 \end{bmatrix} \cdot X_{LDR}$$

$$X_{LDR} = \begin{pmatrix} -814.2986 \\ -0.6256 \\ -1137.7548 \end{pmatrix} + \begin{bmatrix} -0.999132123 & -0.004616436 & -0.041396734 \\ 0.004472887 & -0.999983661 & 0.003559591 \\ -0.04141249 & 0.003371339 & 0.999136447 \end{bmatrix} \cdot X_{ORB}$$

The nominal value for the tilt angle between LDR and ORB frame is 2.5°. Note that the measured value at the moment of writing the present note was 2.69°.

6.2. SOLAR ASPECT ANGLES

6.2.1. Irradiation angles

The irradiation angles α and β, used by the Solar Array Simulator (SAS), are defined as follows:

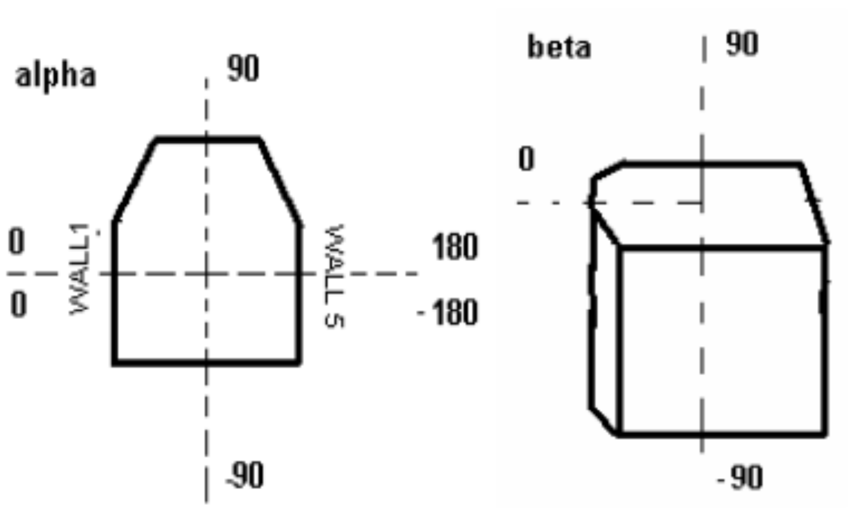


Figure 5 Irradiation angles at SAS.

6.2.2. Transformation from Orbiter SAA to irradiation angles before SDL

Solar illumination on the Orbiter is given via Solar Aspect Angles. The transfer into Lander irradiation angles (α and β) is needed for the solar power computations and has to be performed in 4 steps (excerpt from *RD3*).

6.2.2.1. SAA into Cartesian coordinates

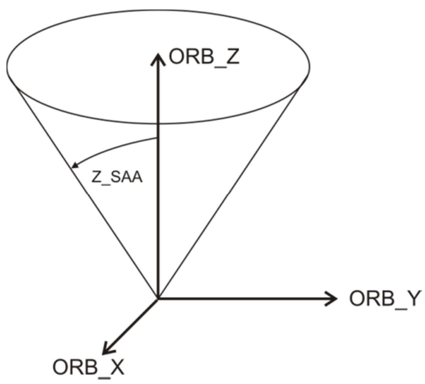


Figure 6 Irradiation angles

The solar aspect angles define cones around the corresponding Orbiter axes.

The direction of Sun illumination is defined by a line from the Orbiter-Ref to the point of intersection of all three cones.

The Cartesian coordinates of this point of intersection are defined as:

$$X_{ORB} = \cos(X\_SAA)$$

$$Y_{ORB} = \cos(Y\_SAA)$$

$$Z_{ORB} = \cos(Z\_SAA)$$

6.2.2.2. Tilting of the Lander attachment plane

The Lander attachment plane is tilted with respect to the Orbiter Y/Z-plane. To align the cartesian coordinates of the SAA into this tilted coordinate system, the following transfer has to be applied (before SDL):

$$X_{LDR} = \begin{pmatrix} -0.999132123 & -0.004616436 & -0.041396734 \\ 0.004472887 & -0.999983661 & 0.003559591 \\ -0.04141249 & 0.003371339 & 0.999136447 \end{pmatrix} * X_{ORB}$$

6.2.2.3. Cartesian coordinates to spherical coordinates

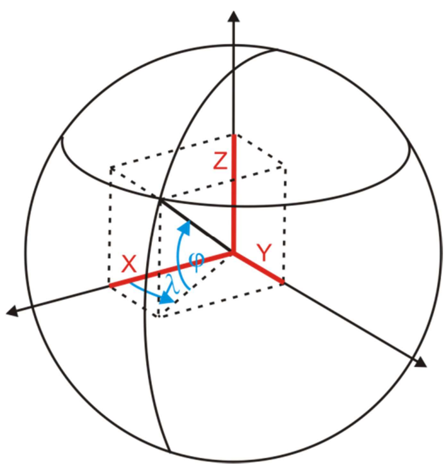
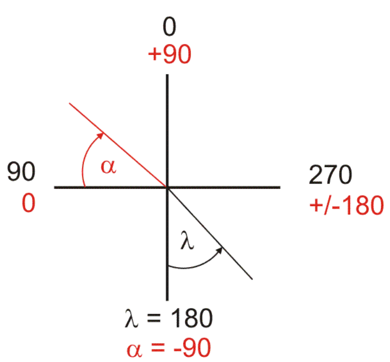


figure 7 cartesian and spherical coordinates

for $x < 0$:	$\lambda = 180° + \arctan(y/x)$
for $x = 0$ and $y < 0$:	$\lambda = 270°$
$y = 0$:	not defined
$y > 0$:	$\lambda = 90°$
for $x > 0$ and $y < 0$:	$\lambda = 360° + \arctan(y/x)$
$y >= 0$:	$\lambda = \arctan(y/x)$
for $x^2 + y^2$ <>0:	$\varphi = \arctan(z/\mathrm{sqrt}(x^2+y^2))$
for $x^2 + y^2 = 0$ and $z < 0$:	$\varphi = -90°$
$z = 0$:	not defined
$z > 0$:	$\varphi = +90°$

6.2.2.4. Spherical coordinates to Lander irradiation angles



for $\lambda = [0,270[$: $\alpha = -\lambda + 90°$

for $\lambda = [270,360[$: $\alpha = 450° - \lambda$

$\beta = \varphi$

6.3. THE LANDING GEAR REFERENCE FRAME - R_{LGF}

6.3.1. Purpose

This frame is used to define the orientation of the Lander with respect to the comet when Philae has already landed.

6.3.2. Description of the Landing Gear Frame

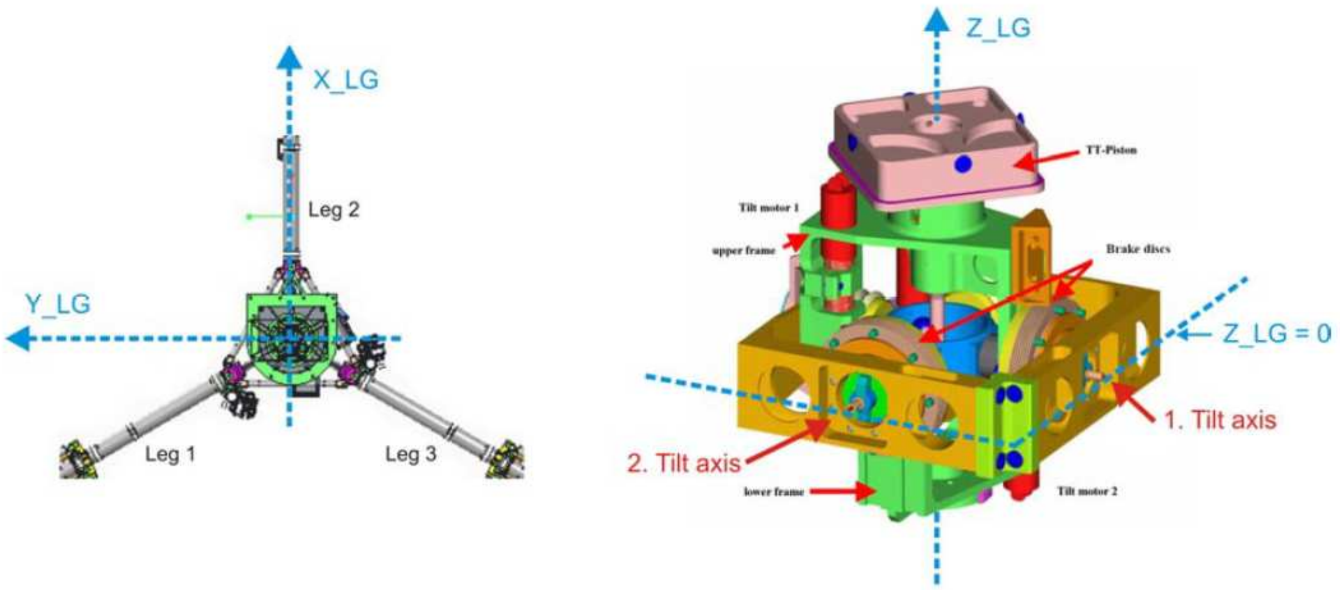


Figure 8 Landing Gear reference Frame.

From RD4, the Landing Gear is defined as follows:

- Origin: In the plane build by the tilting axes in the cardanic joint in the point of intersection of these tilting axes.
- X_{LGF} Parallel to leg 2 (shortest leg), that is to say, pointing to the middle of wall 3 in a direction parallel to the bottom plate when this plate is in its nominal position.
- Z_{LGF} Parallel to the Lander z axis. That is to say, is pointing to the middle of lid plate in a direction parallel to the LG telescopic tube when Philae's body is in its nominal position (same as telescopic tube axis in this position).
- Y_{LGF} axis completes the right handed coordinate system.

6.3.3. Philae's movements w.r.t. R_{LGF}

The Rosetta Lander can move with respect to the landing gear in the following way (RD4):

- A **vertical movement** in the direction of the Z axis. The total difference in length of the tube from cruise position to maximum extension is of 199 mm. However, the bubble is extended during SDL (10 mm) and once in on-comet operation, only an additional 189 mm lifting (and posterior lowering) of the main body w.r.t the landing gear can be achieved. (see below) :

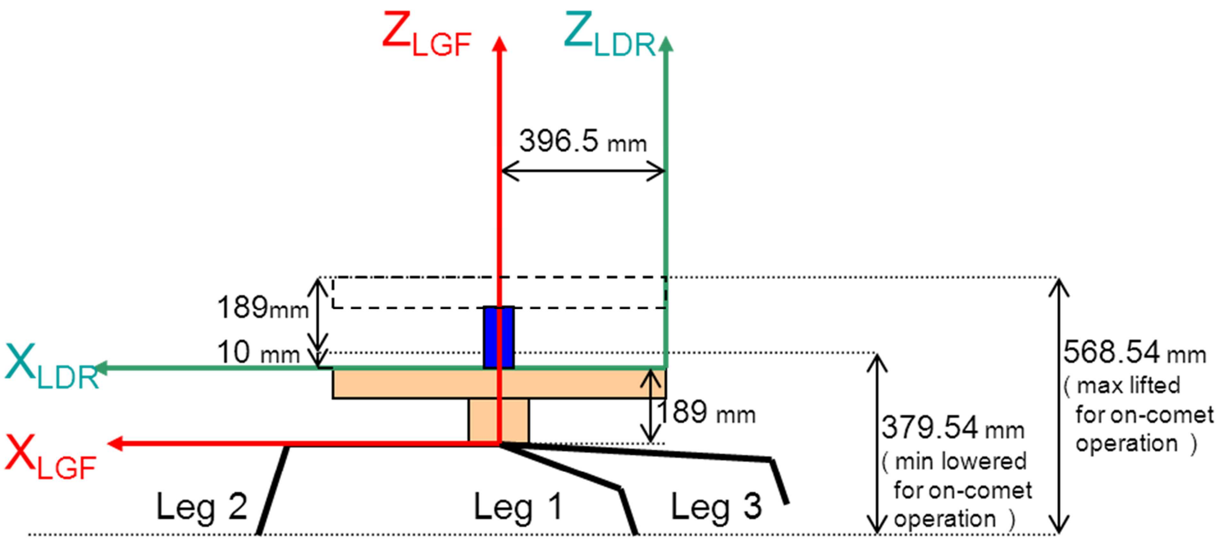


Figure 9 XZ (simplified) view of the Landing Gear and relative position of the X,Z axis of the LDR.

The dimensions shown in **Figure 9** apply only when:

- staying on a flat comet surface
- no foot plate / feet movement during landing has taken place
- no ice screw penetration into the comet surface has happened
- no feet sank into the ground occurred and
- no Lander tilt has been executed.

- A **yaw** or rotation around the Z axis of the Landing Gear Frame. This rotation can be equal to a complete revolution (360 degrees). However, as an operational constraint, a certain minimum lifting of the Lander is necessary to provide the full range of allowed rotation and prevent collision with some parts. The origin of this rotation corresponds to the position where the X axis of the LGF is parallel to the X axis of the LDR. The rotation is considered to be positive if it is done in the counter-clockwise direction (see Figure 10).

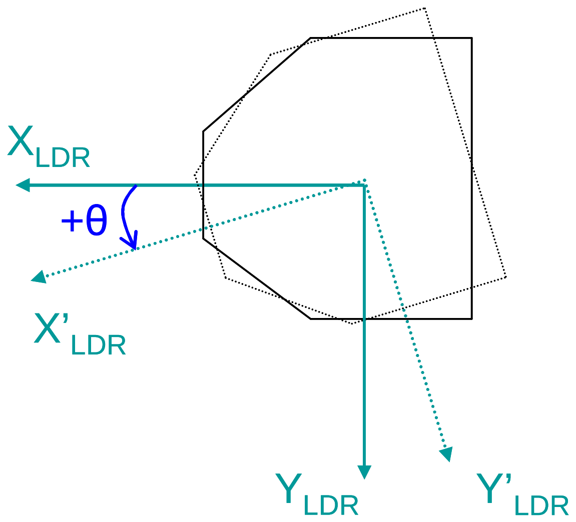


Figure 10 Rotation of Philae with respect to the Z axis of the Landing Gear Frame (viewed from above).

- A **tilt** of the Z axis of ± 3 degrees. The tilting is done with respect to the axis S or T (see **Figure 11**). The tilting causes one side of Philae to be lifted, while the opposite site is lowered w.r.t. the initial position.

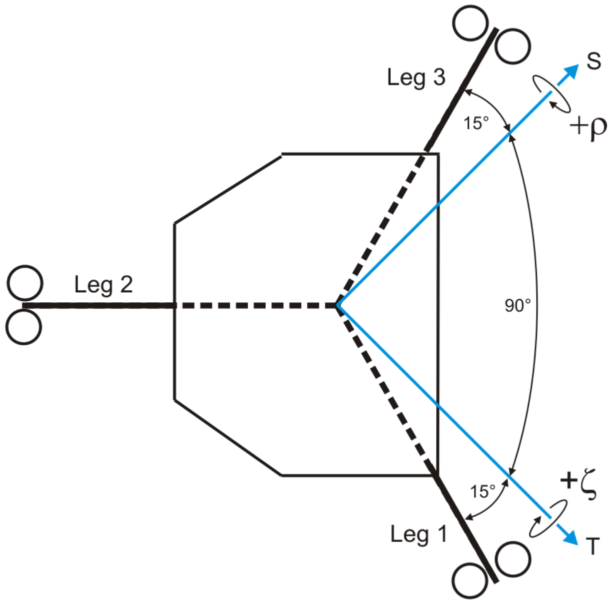


Figure 11 XY view of the Lander on the landing gear, the tilting axis S and T and the positive sense of the rotations around them (viewed from above. Lander in the canonical position in LGF).

6.3.4. Transformation from R_{LGF} to R_{LDR}

6.3.4.1. Canonical position

When Philae is on the canonical position on the Landing Gear, the transformation between Philae Lander Frame and Landing Gear Frame coordinates is as follows (see **Figure 9**)

$$X^0_{LDR} = X_{LGF} + 396.5$$

$$Y^0_{LDR} = Y_{LGF}$$

$$Z^0_{LDR} = Z_{LGF} – 189$$

Once the Landing gear has been deployed and Philae is no longer in cruise configuration (after SDL), the transformation for the Z coordinate changes to:

$$Z^0_{LDR} = Z_{LGF} – 199$$

After the landing, the parameters for the variation in height, rotation around the Z axis of the Landing Gear Frame (angle θ) and tilting of the Z axis of Philae w.r.t. the Z axis of the Landing Gear will have to be commanded (angle σ around axis S and angle ζ around axis T). The commanded values will define the final position of the Lander and the way in which the transformations will take place is detailed in the following subsections.

6.3.4.2. Translation in Z direction

The bubble extension or compression after SDL will be commanded by means of Lander elevation, a height parameter, h, which can take values between 0 and 199 mm (cf. Table 2). The transformation from canonical to extended position is then:

$$X_{LDR} = X^0_{LDR}\,,\quad Y_{LDR} = Y^0_{LDR}$$

$$Z_{LDR} = Z^0_{LDR} – h$$

Before release	h = 0 mm
After landing	10 ≤h≤ 199 mm

Table 2: Lander elevation h according the various moment of the mission.

6.3.4.3. Rotation and tilting

The axis around which each of the possible rotations takes place, as well as the minimum and maximum values of the command parameters are described in the previous subsections. In a general form, the transformation matrix M from a vector V in LGF coordinates to LDR coordinates can be written as the product of three rotation matrix:

$$M(\zeta, \rho, \theta) = [R(T, \zeta)\ R(S, \rho)\ R(Z_{LGF}, \theta)]$$

where R(Z, θ) is the rotation around the Z axis of the LGF of angle θ (Lander Rotation), R(S, ρ) is a rotation matrix of angle ζ around S (Lander tilt around S) and R(T, ζ) is a rotation matrix of angle ρ. around the axis T Lander tilt around T (see A.1.4 for a the explicit rotation matrix representation).

The order used to write the transformation (1 translation and 3 rotation) is important. In the above equation, we first perform lander elevation, then the yaw rotation and finally the rotation around the two tilt angles (T, S).

If the order of transformation was different, the M matrix as written above is not valid any more!

Remark: rotation between T and S are commutative as their rotation axis is independent from each other. This is of course not the case for the other transformations.

6.3.4.4. Final Equation

Therefore, the final equation to be used in order to transform a vector V_{LGF} given in LGF coordinates into a vector V_{LDR} given in LDR coordinates is:

$V_{LDR} = M(\zeta, \rho, \theta)\, V_{LGF} + \begin{pmatrix} 396.5 \\ 0 \\ -199 - h \end{pmatrix}$, where 10 mm ≤ h ≤199 mm depends on the bubble extension.

7. FRAMES LINKED TO THE COMET

7.1. COMET FIXED FRAME – R_{CFF}

7.1.1. Purpose

The Comet Fixed Frame is a non-inertial frame rotating along with the comet. The comet nucleus shape model determines this reference frame. The outgassing production and the values of the gravity field are expressed according to it. Furthermore, this reference system is the comet fixed frame to be used for expressing the information about the surface relevant for the Landing Site Selection (landmarks), as well as for the data exchanges.

7.1.2. Definition

The Comet Fixed Frame is defined as follows (RD5):

- Centred on the centre of masses of the comet.
- Z axis along the rotational axis of the comet and in the direction of the positive pole
- X axis: intersection between the equator of the comet and the prime meridian (established in the shape model)
- Y axis: completes the right hand system

7.1.3. Transformation from EME2000 to R_{CFF}

The transformation from EME2000 to R_{CFF} at a given moment *t* can be done by means of three rotations, once the angles $\alpha_0(t)$, $\delta_0(t)$ and W(t) have been defined:

- Rotation of an angle $\alpha_0(t) + \pi/2$ around $Z_{EME2000}$ axis.
- Rotation of an angle $\pi/2 - \delta_0(t)$ around the new axis X_1
- Rotation of an angle W(t) around the new Z axis.

The angles $\alpha_0(t)$ and $\delta_0(t)$ are the right ascension and declination respectively of the body's positive pole of date relative to the Earth mean equator and equinox of J2000 (EME2000).The third angle, W, is measured along the body's true equator in an easterly direction on the body's surface from the ascending node of the body's equator on the Earth mean equator (see Figure 12).

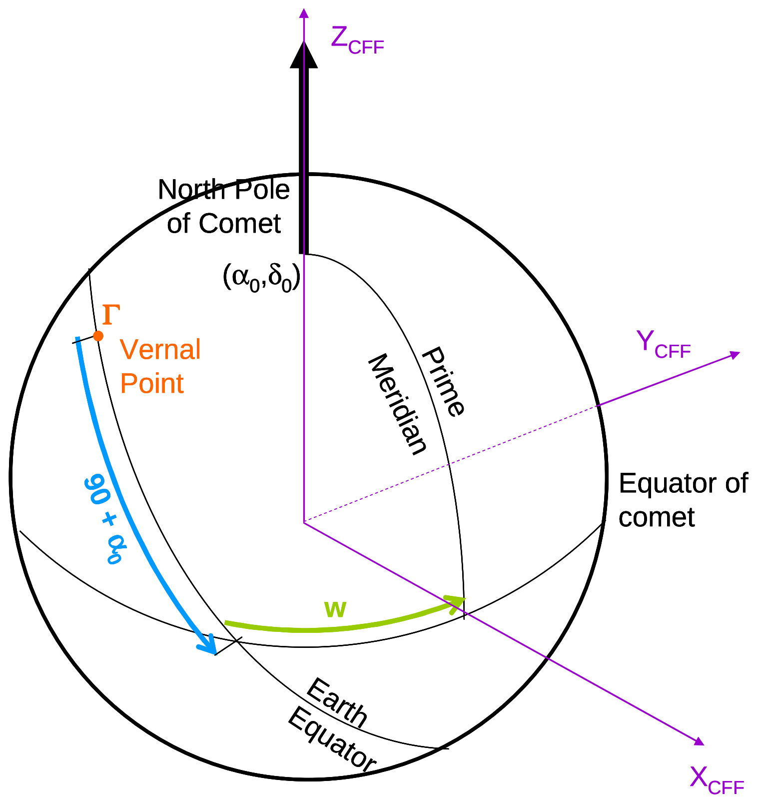


Figure 12 Orientation of the Comet Fixed Frame with respect to EME2000.

The corresponding rotation matrix is (see §A.1.4):

$$M = \begin{bmatrix} -\cos(W)\sin(\alpha)-\sin(W)\sin(\delta)\cos(\alpha) & \cos(W)\cos(\alpha)-\sin(W)\sin(\delta)\sin(\alpha) & \sin(W)\cos(\delta) \\ \sin(W)\sin(\alpha)-\cos(W)\sin(\delta)\cos(\alpha) & -\sin(W)\cos(\alpha)-\cos(W)\sin(\delta)\sin(\alpha) & \cos(W)\cos(\delta) \\ \cos(\delta)\cos(\alpha) & \cos(\delta)\sin(\alpha) & \sin(\delta) \end{bmatrix}$$

In practice, the transformation from R_{CFF} to EME2000 (in both directions) will be done by using the quaternion provided in the file CKIN (file provided by ESOC Flight Dynamics during the comet phase). Excerpt from section §2.5.2 in RD6 :

The attitude of the comet is provided in the comet kinematics file (CKIN). […] The attitude access software returns a quaternion that describes the rotation from inertial frame (EME2000) to the comet fixed frame. The comet fixed frame is defined by the position of landmarks that can be observed in the images taken by the onboard cameras. Based on this definition, the axes of the comet fixed frame will not necessarily coincide with the principal inertia axes.

In this way, not only is the coherence with ESA coordinate frames ensured, but also the computation by the RLGS of the rotation axis of date becomes unnecessary.

7.2. LANDING SITE LOCAL REFERENCE FRAME R_{LSF}

7.2.1. Purpose

This reference frame is one of the reference frames needed to define the orientation of the Lander once on the ground and allows to compute orbital events such as camera dazzling and solar panel illumination.

7.2.2. Definition of the local landing site frame (R_{LSF})

The local landing site frame is defined by:

- Local vertical of the landing site, $\vec{V}_{ls}$ is perpendicular to the horizontal plane tangent to the landing site (as given by the shape model).
- The East axis, $\vec{E}_{ls}$, is the cross product of the comet pole axis with the local vertical to the landing site: $\vec{E}_{ls} = \vec{Z}_{iner} \wedge \vec{V}_{ls}$.
- The North axis is such that the trihedron ($\vec{V}_{ls}$, $\vec{E}_{ls}$, $\vec{N}_{ls}$) is direct, $\vec{N}_{ls} = \vec{V}_{ls} \wedge \vec{E}_{ls}$

7.2.3. Representation

This scheme assumes spherical coordinates for the localisation of the landing site.

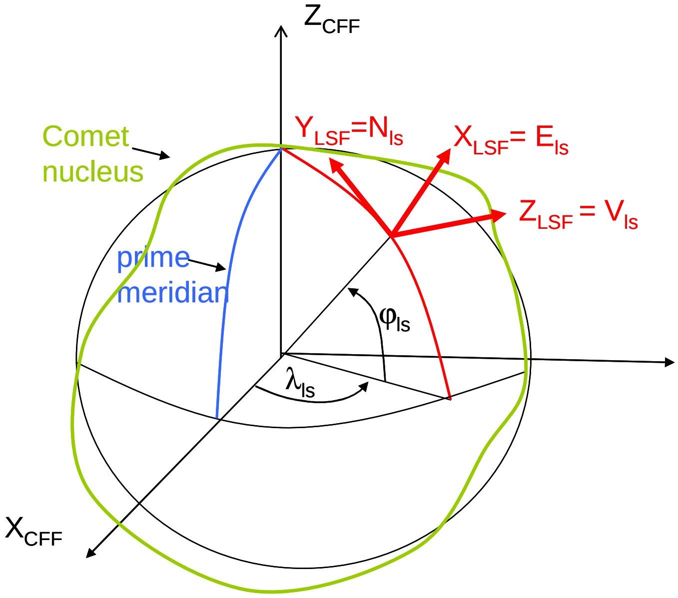


Figure 13 The landing site reference frame (λ_{ls} = longitude, φ_{ls}=latitude of landing site)

Note that the Local landing Site reference frame depends on the shape model used at the moment of the comet operations (for the computation of the local tangent plane) as well as on convention taken for the comet rotation axis (Z axis defined for the CFF).

7.2.4. Remarks

Definition of the azimuth: The azimuth angle is defined on the plane tangent to the landing site. The origin of azimuths is the North direction as defined in § 7.2.2, increasing eastwards.

Definition of local landing site: The definition of the plane tangent to the impact point can seem *fuzzy* when the shape model has a very good resolution and Philae's body overlaps several different planes. However, the LSF has to be considered as a theoretical reference frame, for which the Z_{LSF} is taken as the local vertical given by the shape model (both for a rough or a DTM approximation) at the punctual (latitude, longitude) identified as landing site after touchdown.

Singularities of the LSF: The local Landing Site reference Frame is not defined at the poles, East direction being undefined. In this case, it can be replaced by a reference system whose Z axis corresponds to the local vertical, the X axis is tangent to the prime meridian, and the Y axis is obtained such that the system is a positive trihedron.

7.2.5. Transformation from LSF to LDR

The transformation from LSF to LDR is a complicated problem that will be treated in what we call *attitude determination on the ground*. The output of this process will be a quaternion representing the deviation of the three LDR axis w.r.t. the local vertical and the local East direction. Note that we call local site vertical to the direction perpendicular to the landing site tangent plane as given by the shape model used at the moment of the landing. However, the local vertical (Z_{LSF}) may not correspond to the actual direction perpendicular to the comet surface, neither be aligned with the Z axis of the Lander (see **Figure 14**).

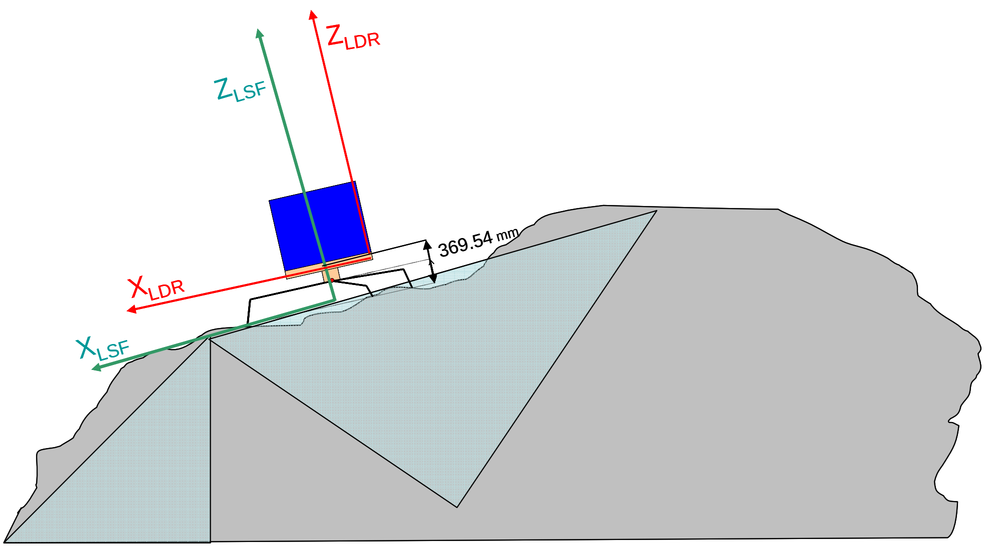


Figure 14 **Representation of the position of the Lander on the comet and the shape model (triangles).**

In addition to the attitude change between LSF and LDR, a translation in the X and Z directions of the LDR, going from surface level (origin of the LSF frame) to the origin of coordinates of the LDR is also necessary (see §6.1.3). So, if V_{LSF} is a vector expressed in the LSF coordinates and M is the rotation matrix from LSF to LDR, the following equation shows how to obtain V in LDR frame:

$$V_{LDR} = M\, V_{LSF} + \begin{pmatrix} 396.5 \\ 0 \\ -369.54 - h \end{pmatrix}$$

where h, 10mm ≤ h ≤199, mm is the vertical extension of the landing gear bubble for on-comet operations. Note that this equation applies only when neither rotation nor tilting has been commanded to the Lander w.r.t. the LGF.

Note: The phase referred to as azimuth in LDR frame is the angle between the X_{LDR} and the LSF North direction.

7.2.6. Transformation from LSF to CFF

Given a landing site, P, with spherical coordinates (r_{loc}, λ, φ), the shape model file allows for the computation of the vector normal to the plane which is tangent to the surface of the comet at P. Let *u* = (u_1, u_2, u_3) be this vector in CFF coordinates.

The LSF frame is defined as follows:

- Z_{LSF} axis corresponds to vector *u* (local vertical).
- X_{LSF} can be computed as : $X_{LSF} = Z_{iner} \wedge Z_{LSF}$ where Z_{iner} is the rotation axis of the comet.

In CFF coordinates, $Z_{iner} = Z_{CFF} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, so $X_{LSF} = Z_{CFF} \wedge u = \begin{pmatrix} -u_2 \\ u_1 \\ 0 \end{pmatrix}$

- Y_{LSF} completes the positive trihedron, so $Y_{LSF} = Z_{LSF} \wedge X_{LSF} = \begin{pmatrix} -u_1 u_3 \\ -u_2 u_3 \\ u_1^2 + u_2^2 \end{pmatrix}$

Therefore, the rotation matrix M from LSF to CFF is:

$$M = \begin{bmatrix} -u_2 & -u_1 u_3 & u_1 \\ u_1 & -u_2 u_3 & u_2 \\ 0 & u_1^2 + u_2^2 & u_3 \end{bmatrix}$$

The origin of coordinates of the LSF reference frame is the landing site P. Therefore, a translation is needed to complete the transformation from LSF to CFF (see A.3).

If a_{LSF} is a vector given in LSF coordinates, then a_{CFF} is the same vector expressed in CFF :

$$a_{CFF} = \begin{bmatrix} -u_2 & -u_1 u_3 & u_1 \\ u_1 & -u_2 u_3 & u_2 \\ 0 & u_1^2 + u_2^2 & u_3 \end{bmatrix} a_{LSF} + \begin{pmatrix} r_{loc} \cos\varphi \cos\lambda \\ r_{loc} \cos\varphi \sin\lambda \\ r_{loc} \sin\varphi \end{pmatrix}$$

ANNEX A CONVENTIONS

A.1. TRANSFORMATION MATRIX AND ROTATION MATRIX

By convention, the transformation matrix TM_{12} from an initial frame R_1 to a final frame R_2 with the same origin is defined by the relation $X_2 = TM_{12}.X_1$, where X_1 and X_2 are the components of the vector X expressed respectively in the frames R_1 and R_2. We are interested in particular in the case of transformations from one reference frame to another. The transformation matrix is then a rotation, which can be expressed as detailed in the following sections.

A.1.1. Rotation around X axis

If the rotation of angle θ_1 around the X axis (in an anti-clockwise direction when looking toward the origin), is applied to a vector M_i of components (x_i, y_i, z_i), the vector M_f obtained has the components (x_f, y_f, z_f):

$$\begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} = R_1(\theta_1).\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \quad \text{with } R_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

A.1.2. Rotation around Y axis

If the rotation of angle θ_2 around the Y axis is applied to a vector M_i of components (x_i, y_i, z_i), the vector M_f obtained has the components (x_f, y_f, z_f) :

$$\begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} = R_2(\theta_2).\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \quad \text{with } R_2(\theta_2) = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

A.1.3. Rotation around Z axis

If the rotation of angle θ_3 around the Z axis is applied to a vector M_i of components (x_i, y_i, z_i), the vector M_f obtained has the components (x_f, y_f, z_f):

$$\begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} = R_3(\theta_3).\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \quad \text{with } R_3(\theta_3) = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A.1.4. Generic rotation around axis u=(u_x, u_y, u_z)

If the rotation of angle θ around an axis u of components (ux, uy, uz) is applied to a vector M_i of components (x_i, y_i, z_i), the vector M_f obtained has the components (x_f, y_f, z_f):

$$\begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} = R(u,\theta)\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \text{ with}$$

$$R(u,\theta)=\begin{bmatrix} \cos\theta+u_x^2(1-\cos\theta) & u_yu_x(1-\cos\theta)-u_z\sin\theta & u_zu_x(1-\cos\theta)+u_y\sin\theta \\ u_yu_x(1-\cos\theta)+u_z\sin\theta & \cos\theta+u_y^2(1-\cos\theta) & u_yu_z(1-\cos\theta)-u_x\sin\theta \\ u_zu_x(1-\cos\theta)-u_y\sin\theta & u_yu_z(1-\cos\theta)+u_x\sin\theta & \cos\theta+u_z^2(1-\cos\theta) \end{bmatrix}$$

A.1.5. Properties of the rotation matrix

Note that when a rotation of angle θ around a given axis is represented in a matrix R, the following properties are satisfied:

$$R(-\theta) = - R(\theta)$$

$$R(\theta)^{-1} = R(\theta)^T$$

A.1.6. Combination of rotations

Let's consider a rotation that transforms (X_i, Y_i, Z_i) frame into (X_f, Y_f, Z_f), then $\begin{pmatrix} x_f \\ y_f \\ z_f \end{pmatrix} = R.\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}$.

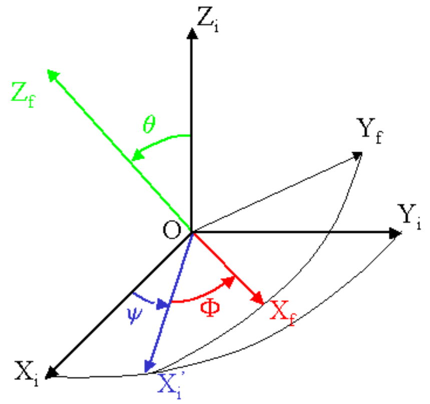


Figure A. 1 Combination ofthree rotations

A.1.6.1. Euler angles expression

The rotation may be decomposed into three rotations respectively around Z_i, $X_i^{'}$ and Z_f axis of respective angles ψ, ϕ, θ. $\psi = (OX_i, OX_i^{'})$, $\theta = (OZ_i, OZ_f)$ with $0^\circ \le \theta \le 180^\circ$ and $\phi = (OX_i^{'}, OX_f)$.

$$R = R_3(\phi).R_2(\theta).R_1(\psi) = \begin{bmatrix} \cos\phi\cos\psi - \sin\phi\sin\psi\cos\theta & \cos\phi\sin\psi + \sin\phi\cos\psi\cos\theta & \sin\phi\sin\theta \\ -\sin\phi\cos\psi - \cos\phi\sin\psi\cos\theta & -\sin\phi\sin\psi + \cos\phi\cos\psi\cos\theta & \cos\phi\sin\theta \\ \sin\psi\sin\theta & -\cos\psi\sin\theta & \cos\theta \end{bmatrix}$$

A.2. QUATERNIONS

Quaternions will be used to describe the vehicle attitude w.r.t and inertial frame.

It is possible to associate to a rotation, defined by its axis u (u_x,u_y,u_z) and angle θ, a quaternion $q=q_0 + q_1 i + q_2 j + q_3 k$, with:

$$q_0 = \cos(\theta/2)$$

$$q_1 = u_x \sin(\theta/2)$$

$$q_2 = u_y \sin(\theta/2)$$

$$q_3 = u_z \sin(\theta/2)$$

i, j and k are imaginary numbers satisfying:

$$i^2=j^2=k^2=-1$$

$$ij=-ji=k$$

$$jk=-kj=i$$

$$ki=-ik=j$$

From Euler's rotation theorem we know that any rotation matrix R can be equivalently expressed by a quaternion q(u,α). That is to say that any rotation in space can be expressed as a basic rotation of angle α around an axis *u*.

Moreover, the attitude of a satellite w.r.t. an inertial reference frame is usually given by means of quaternion at each given epoch t. This mathematical representation has obvious advantages and the algebra associated to quaternion allows for attitude conversions and analysis with no need for matrix representation whatsoever (see for instance RD7).

A.3. SPHERICAL COORDINATES

The point P can be defined thanks to spherical coordinates: the radial distance r, the latitude φ and the longitude λ with:

$$r \geq 0$$
$$-\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}$$
$$0 \leq \lambda \leq 2\pi$$

as illustrated in**Erreur ! Source du renvoi introuvable.**.

The transformation which allows us to obtain Cartesian coordinates from spherical coordinates is:

$$x = r\cos\varphi\cos\lambda$$
$$y = r\cos\varphi\sin\lambda$$
$$z = r\sin\varphi$$

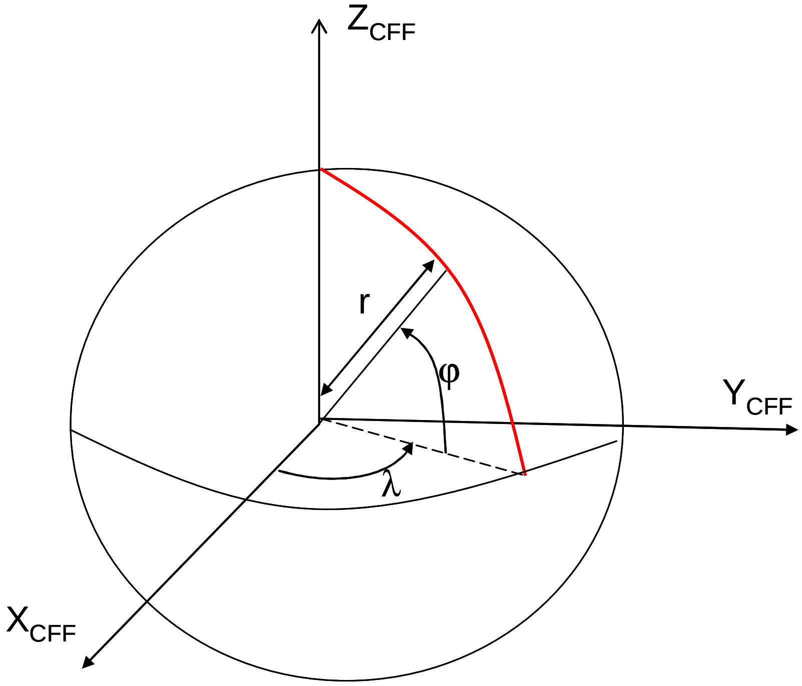


Figure A. 2 Spherical coordinates

A.4. LOCAL ORBITAL FRAME

A.4.1. Purpose

The frame is used for the analysis of trajectories around objects. It could be used in the case Comet/ Sun or Satellite/Comet or even Satellite/Sun

A.4.2. Definition of the local orbital frame (R_{QSW})

The local orbital frame is, of course, linked to the second body's ephemeris around the first body. Its orientation, expressed in the EME2000 frame depends on the date of computation. The W vector is along the orbit momentum, it is perpendicular to the trajectory plane. The following relation gives its components: $W = R \wedge V$, where R stands for the position vector in EME2000 and V its velocity vector. The vector W is approximately constant (orbit angular momentum).

The Q vector is along the primary-secondary-direction, from the primary toward second. Finally S vector makes the QSW trihedron a direct trihedron. This frame is always well defined (i.e. the Sun direction is never perpendicular to the plane of orbital motion).

A.4.3. Rules

As the Q vector goes along the bodies direction and secondary body is moving around the primary, its components change with the time.

A.4.4. Representation

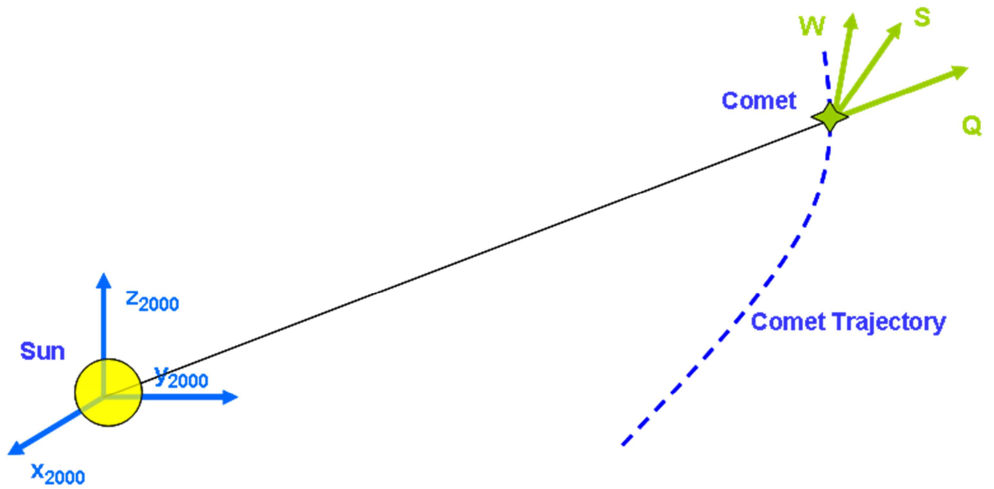


Figure A. 3 Local Orbital Reference Frame

A.4.5. Transformation from QSW to EME2000

Given a vector in QSW orbital frame, V_{QSW}, at a given epoch *t* the transformation which gives the vector $V_{EME2000}$ in EME2000 is:

$$V_{EME2000} = M(t)\, V_{QSW} + r(t)$$

where M(t) is the matrix whose columns are the unit vectors in the directions of Q, S and W respectively. Let:

- $r(t) = (r_1, r_2, r_3)$ be the position at time *t* in EME2000 coordinates, v(t) the velocity in the same reference frame, $v(t) = (v_1, v_2, v_3)$ and α(t) the angle between r(t) and v(t)
- u(t) be the position of the main body in EME2000 coordinates at time t, $u(t) = (u_1, u_2, u_3)$.

Then we can find the unit vectors in the directions of W, Q and S as follows,

$$W = \frac{r \wedge v}{\|r \wedge v\|} = \frac{1}{\|v\|\|r\|\sin\alpha}(r_2 v_3 - r_3 v_2, r_3 v_1 - r_1 v_3, r_1 v_2 - r_2 v_1) = (w_1, w_2, w_3)$$

$$Q = \frac{r - u}{\|r - u\|} = \frac{1}{\|r - u\|}(r_1 - u_1, r_2 - u_2, r_3 - u_3) = (q_1, q_2, q_3)$$

$$S = \frac{W \wedge Q}{\|W \wedge Q\|} = \frac{1}{\sin\beta}(w_2 q_3 - w_3 q_2, w_3 q_1 - w_1 q_3, w_1 q_2 - w_2 q_1) = (s_1, s_2, s_3)$$

where β(t) is the angle between Q (position of the second body in a main body centred J2000 coordinates) and W (the angular momentum of the orbit) at time *t*. Note that, for the sake of simplicity, the time dependence has been omitted in the equations above.

Then we have, $M(t) = \begin{bmatrix} q_1 & s_1 & w_1 \\ q_2 & s_2 & w_2 \\ q_3 & s_3 & w_3 \end{bmatrix}$